MODELLING ICE THICKNESS

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>
> **Peter Wadhams, Professor of Ocean Physics, University of Cambridge**

The most visible sign of the change in Arctic sea ice is the shrinking of the area it covers, which can be seen from satellites. But scientists believe that the thinning of the ice is even more important when it comes to the future of the ice caps – they believe that the Arctic ice cover will break up and disappear through thinning, rather than shrinkage. If current trends continue this will happen as early as 2030, leaving no ice at all in summer.

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The Catlin Arctic Survey ice team moving on thin ice

But how can we predict the impact of climate change on ice thickness? The most important first step is to understand how ice grows in general. When left alone in more or less constant air and water temperatures, will a piece of ice that's floating on sea water just keep on getting thicker and thicker? Or will it eventually stabilise at a constant thickness?

To find out, a scientist working at an Arctic research station has observed a layer of ice forming on sea water for a period of two weeks. It's the Arctic winter and there is no large change in air or water temperature from day to day. Every day she measured the thickness of the ice. These are her measurements:

Time	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Thickness	1.3cm	1.8cm	2.3cm	2.6cm	3cm	3.2cm	3.4cm
Time	Day 8	Day 9	Day 10	Day 11	Day 12	Day 13	Day 14
Thickness	3.7cm	4cm	4.1cm	4.3cm	4.5cm	4.7cm	4.8cm

Question 1: Plot the data points in a graph. What does the pattern of the points tell you about the rate of ice growth?

Question 2: Can you think of a mathematical function $f(x)$ with a shape similar to that formed by your data points?

Question 3: Plot the functions $f(x) = \sqrt{x}$ and $f(x)=1.5\sqrt{x}$ in your coordinate system. Discuss how they relate to your data points.

Based on her data, the scientist has decided to model the growth in ice thickness over time by the function

$$h(t) = c\sqrt{t}.$$

Here h denotes the thickness of the ice measured in centimetres, t is time measured in days, and c is a constant.

Question 4: Using the data, can you estimate what value c should take?

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The Catlin Arctic Survey ice team use ice drills to measure ice thickness.

Question 5: The scientist performs a similar experiment in the Arctic summer, when the temperature of air, despite still being below freezing, is a lot warmer than during the winter. How would you expect the constant c to change?

The rule we have just developed, which states that the growth of ice thickness is proportional to the square root of time, is known as the *Stefan law*, after the Austrian scientist Josef Stefan, who developed it in the 1890s.

In this example our scientist came up with the model by carefully looking at her data and finding a mathematical function that matches the data well. However it's possible to derive a very similar model directly from the laws of physics that govern the way heat is conducted in different materials. See the *Plus* magazine article *Maths and climate change: The melting Arctic* (http://plus.maths.org/issue46/features/wadhams/index.html) for more information.

This simple model only tells us what happens to ice if the air and water temperatures remain constant, and it ignores many other factors that impact on sea ice, for example the way it moves around with sea currents or is driven by the wind. It therefore does not predict the long term future of the Arctic ice cap. However, simple models like this one form the basic ingredients of the large-scale models used by scientists to predict the effects of climate change.