MAPPING THE ARCTIC WITH MATHEMATICS

The aim of the Catlin Arctic Survey is to gather information to better understand the future of the Arctic sea ice cover. The route taken by the ice team has been carefully planned with the help of scientists using a detailed map of the Arctic – but how do you make such a map?

Question 1: Would you take the map below on a trip to the Arctic? What do you notice about the relative sizes of countries on this map?

* *Answer: No. The North Pole is not actually on this map at all and the area of countries is greatly exaggerated as you move towards the Poles. For example, Greenland appears bigger than Africa, but in reality its area is actually over 14 times smaller than that of Africa.*
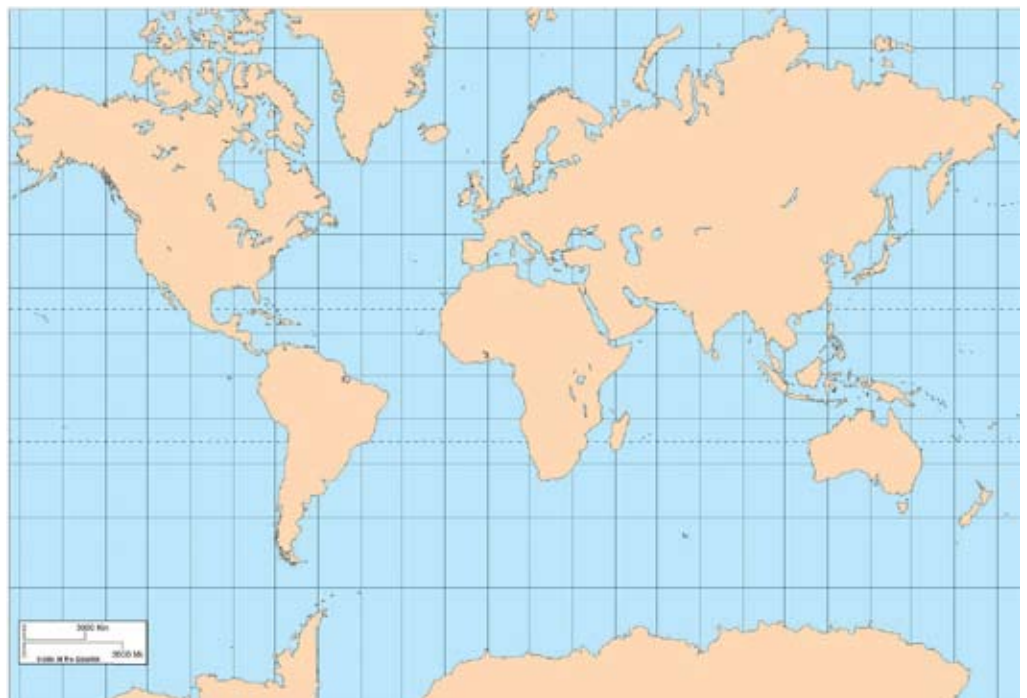

Image © Alabama Maps http://alabamamaps.ua.edu

Any map is a two-dimensional representation of the Earth, which is a sphere. Any such projection comes with some degree of distortion. On this projection, called the *Mercator projection*, area is exaggerated as you move towards the Poles, and the Poles themselves are missing.

"**Good navigation tools are essential – our life depends on them!**"

Pen Hadow Expedition Leader

Here is a simplified description of how the Mercator projection works: place the spherical Earth into a cylinder of the same diameter, with the equator touching the cylinder. Now imagine placing a light at the centre of the Earth. Each point *x* on the surface of the Earth will have a 'shadow' *y* on the cylinder – *y* is the point where the straight line that connects *x* and the centre of the Earth hits the cylinder. In this way you can *project* the Earth onto the cylinder. Now cut the cylinder open (you can choose along which line to do this, but usually it is cut along the 180 degree meridian) – that's the Mercator projection!
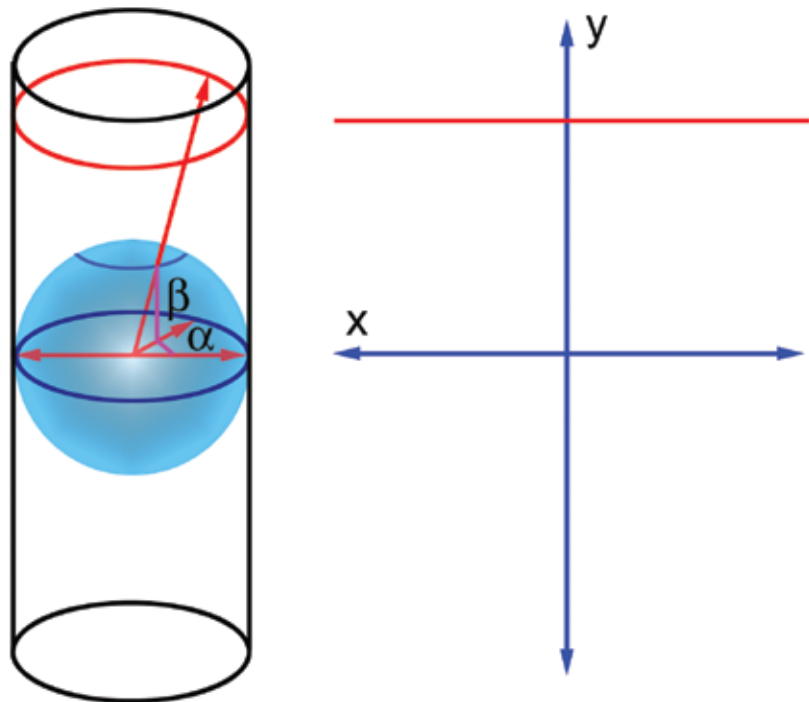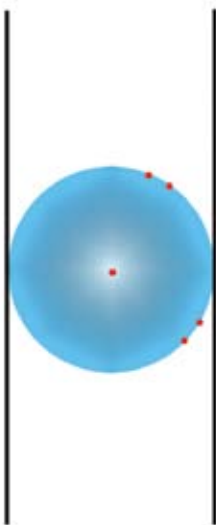

The mercator projection projects the Earth on a cylinder and then cuts it open

This is how discs of equal area are distorted:
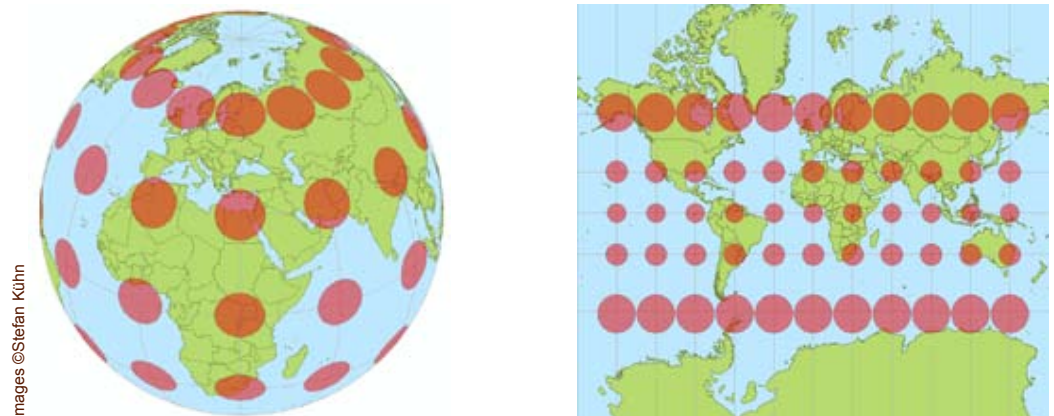
Question 2: The diagram to the left shows a 2-dimensional representation of the projection set-up, viewed from the side. Find the projections of the points marked in red. What happens to the Geographic North Pole? Does this diagram explain the distortion? How tall a cylinder do you need to project the entire globe?

Answer: The North Pole has no projection because the straight line connecting the centre of the Earth and the North Pole never hits the cylinder at all. There is distortion in the map because the projection amplifies distances between points near the Poles. You'd need an infinitely tall cylinder to project the entire globe. So to construct a useful map, you need to decide on two 'cut-off latitudes': points on the Earth which lie further North or South than these latitudes will not be represented on the map.
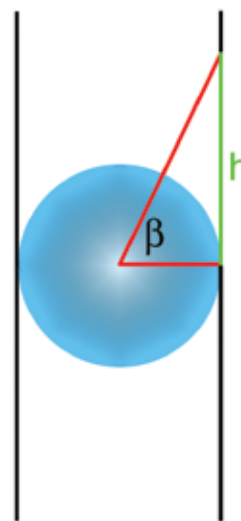
Question 3: In the diagram to the right the angle ß describes the latitude of the point that has been marked. Assume that the radius of the Earth is **1** unit. Express the height *h* in terms of trigonometric functions of the unit circle.

Answer: $h = \tan(\beta)$.

Each point on the globe is given by its longitude α and latitude β. These are angles measured in degrees. The projection above maps each point to the point on the map with co-ordinates

$$x = \alpha$$
$$y = R\,tan(\beta).$$

The number R is simply a scaling factor: if you want a very large map, you make R very large, and if you want a very small map, you make it small.

Question 4: Cairo has a latitude of approximately 30 degrees, London has a latitude of approximately 51 degrees, and Spitzbergen has a latitude of approximately 80 degrees. Setting R=1, use the formula above to work out their y-coordinate on a map. What do you notice? Can we use this projection to map the Poles?

Answer: Cairo has y-coordinate 0.577, London has y-coordinate 1.234 and Spitzbergen has y-coordinate 5.671. The y-coordinates become larger rapidly as you go North. The y-coordinates of the Poles are not defined.

Question 5: Sketch the graph of the function y=tan(ß) for ß ranging from -90 degrees to 90 degrees. How does this graph explain the distortion on the map?

Answer: The distortion is reflected by the fact that the slope of the graph becomes larger and larger as you approach -90 degrees and 90 degrees.

Question 6: So the Mercator projection is useless for anyone wanting to explore the Arctic. Can you come up with another projection which represents the Arctic more accurately?

＊ *Answer: This is an open-ended question which is discussed in the accompanying article, but some possible projections include:*

＊ *Transverse Mercator projection, which simply rotates the sphere within the cylinder, so that the North and South Poles touch the cylinder.*
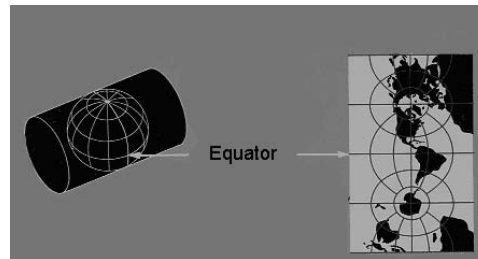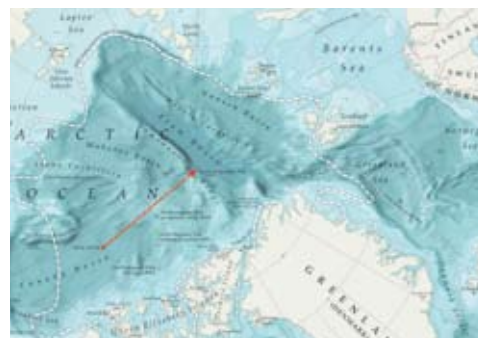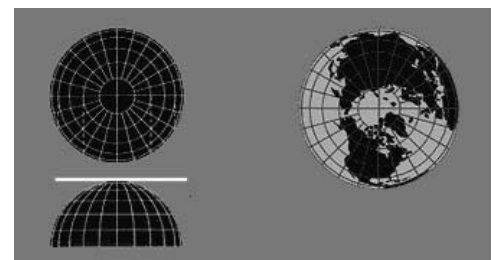
＊ *Stereographic projection, projecting a hemisphere onto a flat plane from a point on the Earth. For a map of the Arctic one would use the South Pole as the point to project from.*
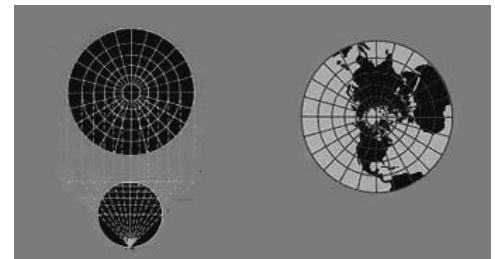
＊ *Gnomonic projection projecting a hemisphere onto a flat plane from the centre of the Earth.*
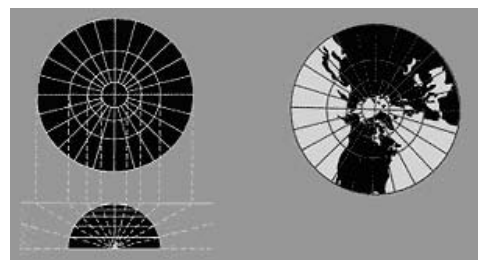
＊ *Orthographic projection, projecting a hemisphere onto a flat plane from a point at infinity.*

＊ *The map showing the intended route of the ice team.*